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Lecture Notes

B. Sc. Mathematics (H)

I - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	1	2	3	**Straight Line**	**Dr. D. K. Yadav**	23

II - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	2	4	2	**Orthogonal Trajectories**	Dr. D. K. Yadav	27

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Part-1	Part-2
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B. Sc. Maths (H)	B. Sc. Maths (H)
Straight Line	**Orthogonal Trajectories**

Part-1

Straight Line

Symmetric Form: The equation of the line passing through a given point (α, β, γ), whose direction cosines are l, m, n is given by

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} (= r)$$

where A is the point whose co-ordinates are (α, β, γ) and P(x, y, z) be any arbitrary point on the line such that AP = r.

Note:

1. From above relation we get

$$x = \alpha + lr, y = \beta + mr, z = \gamma + nr$$

which are the general co-ordinates of any point on the straight line at a distance r from the point (α, β, γ).

2. A line parallel to x-axis and passing through the point (α, β, γ) is $y = \beta, z = \gamma$.

A line parallel to y-axis and passing through the point (α, β, γ) is $x = \alpha, z = \gamma$.

A line parallel to z-axis and passing through the point (α, β, γ) is $x = \alpha, y = \beta$.

3. If the line is perpendicular to the plane ax + by + cz + d = 0, then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$.

Line Through two Points:

The equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Putting each one equal to α, we obtain the general co-ordinates of any point on the line in terms of the parameter α.

Example-1: Find the co-ordinates of the point of intersection of the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

with the plane 2x – 2y – z = 7.

Solution: Let

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} = r$$

Therefore

$$x = 2r + 1,\ y = -3r + 2,\ z = 4r - 3 \qquad (1)$$

Given that

$$2x - 2y - z = 7 \qquad (2)$$

Solving (1) and (2), we get r = 1. Thus the point of intersection is [from (1)] (3, -1, 1).

Example-2: Find the equation of the plane through the line 3x – 4y + 5z = 10, 2x + 2y – 3z = 4 and parallel to x = 2y = 3z.

Solution: The equation of the plane through the intersection of the planes 3x – 4y + 5z = 10 and 2x + 2y – 3z = 4 is

$$(3x - 4y + 5z - 10) + M\,(2x + 2y - 3z - 4) = 0 \qquad (1)$$

Which implies

$$(3 + 2M)\,x + (2m - 4)\,y + (5 - 3M)\,z - (10 + 4M) = 0 \qquad (2)$$

If this line is parallel to x = 2y = 3z i.e., $\frac{x}{1} = \frac{y}{1/2} = \frac{z}{1/3}$, then we must have

(3 + 2M) 1 + (2m – 4) (1/2) + (5 – 3M) (1/3) = 0

which implies M = -4/3.

Putting the value of M in (1) we get

x – 20 y + 27 z = 14

which is the required equation of the plane.

Reference Book: Solid Geometry 3D- Lalji Prasad

Part-2

Orthogonal Trajectories

Trajectory: A curve which cuts every member of a given family of curves according to some definite law is called a trajectory of the family.

Orthogonal Trajectory: A curve which cuts every member of a given family of curves at right angles is called an orthogonal trajectory of the family.

Orthogonal Trajectories: Two families of curves are said to be orthogonal if every member of either family cuts each member of the other family at right angles.

Example: In the two-dimensional problems in the flow of heat, the lines of heat flow in a body are everywhere perpendicular to the isothermal curves or loci of points at the same temperature.

Working Rule to Find the Equation of Orthogonal Trajectories:

For Cartesian Curves f(x, y, c)=0:

Let f(x, y, c)=0 ….. (1)

(i) Differentiate (1) and eliminate the arbitrary constant c between (1) and the resulting equation. That gives the differential equation of the family (1). Let it be

$$F\left(x, y, \frac{dy}{dx}\right) = 0 \qquad \text{…. (2)}$$

(ii) Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$.

(iii) The differential equation of the orthogonal trajectory is $F\left(x, y, -\frac{dx}{dy}\right)=0$ … (3)

(iv) Solve (integrate) (3) to get the equation of the required orthogonal trajectory.

For Polar Curves f(r, θ, c)=0:

Let f(r, θ, c)=0 ….. (1)

(i) Differentiate (1) and eliminate the arbitrary constant c between (1) and the resulting equation. That gives the differential equation of the family (1). Let it be

$$F\left(r, \theta, \frac{dr}{d\theta}\right)=0 \quad \text{…. (2)}$$

(ii) Replace $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$.

(iii) The differential equation of the orthogonal trajectory is $F\left(r, \theta, -r^2\frac{d\theta}{dr}\right)=0$… (3)

(iv) Solve (integrate) (3) to get the equation of the required orthogonal trajectory.

Examples for Exercise

1. Find the orthogonal trajectories of the family of

(i) hyperbolas xy=c, (ii) parabolas $y^2=4ax$,

(iii) parabolas $y=ax^2$, (iv) semi-cubical parabolas $ay^2=x^3$.

2. Show that the family of parabolas $x^2=4a(y+a))$ is self-orthogonal.

3. Find the orthogonal trajectories of the family of coaxial circles $x^2+y^2+2gx+c=0$, g being the parameter.

4. Prove that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, λ being the parameter, is self-orthogonal.

5. Find the orthogonal trajectories of the family of following curves:

(i) $r=a(1+\cos\theta)$ (ii) $r^n \sin n\theta=a^n$ (iii) $r^2=a^2\cos 2\theta$ (iv) $r^n=a^n\cos n\theta$ (v) $r=\frac{2a}{1+\cos\theta}$.

Answers

1. (i) $x^2-y^2=c$ **(ii)** $2x^2+y^2=c$ **(iii)** $x^2+2y^2=c$ **(iv)** $2x^2+3y^2=c$

3. $x^2+y^2+2fy-c=0$

5. (i) $r=c(1-\cos\theta)$ **(ii)** $r^n\cos n\theta=c^n$ **(iii)** $r^2=c^2\sin 2\theta$ **(iv)** $r^n=c^n\sin n\theta$ **(v)** $r=\frac{2c}{1-\cos\theta}$.