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Lecture Notes

B. Sc. Mathematics (H)

~~I - Year~~

~~Sub~~	~~Year~~	~~Paper~~	~~Unit~~	~~Topic~~	~~Author~~	~~Lec. S. N.~~
~~Maths~~	~~1~~	~~2~~	~~2~~	~~Xxxxxx~~	~~Dr. D. K. Yadav~~	~~21~~

II - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	2	4	2	**Exact Differential Equations**	Dr. D. K. Yadav	25

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~~Part-1~~	Part-2
~~Paper-II~~	Paper-IV
~~B. Sc. Maths (H)~~	B. Sc. Maths (H)
~~Xxxxx~~	**Exact Differential Equations**

Exact Differential Equation

Exact Differential Equation:

A differential equation of first order and first degree Mdx+Ndy=0 is said to be exact if there exists a function f(x, y) such that df=Mdx+Ndy i. e. $\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy=Mdx+Ndy$.

The general solution is then given by f(x, y)=C.

Note: An exact differential equation can always be derived from its general solution directly by differentiating, without any subsequent multiplication, elimination, etc.

Necessary and Sufficient Condition for a Differential Equation to be Exact:

The necessary and sufficient condition for Mdx+Ndy=0 to be exact is $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$.

If the given differential equation is exact its general solution is given by

$$\int_{y=cons\tan t} Mdx+\int N(only\ those\ terms\ in\ N\ not\ involving\ x)dy=C$$

Examples for Exercise

Solve the following differential equations:

1. $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ [Ans: $x+ye^{x/y}=c$]

2. $y\sin 2xdx-(y^2+\cos^2 x)dy=0$ [Ans: $3y\cos 2x+2y^3=c$]

3. $\{y\sin(xy)+xy^2\cos(xy)\}dx+\{x\sin(xy)+x^2y\cos(xy)\}dy=0$ [Ans: $xy\sin(xy)=c$]

4. $y\sin 2xdx-(y^2+\cos^2 x)dy=0$ [Ans: $y\cos 2x+(2/3)y^3+y=c$]

5. $\{y(1+1/x)+\cos y\}dx+(x+\log x-x\sin y)dy=0$ [Ans: $xy+y\log x+x\cos y=c$]

6. $y\sin 2xdx-(1+y^2+\cos^2 x)dy=0$