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Lecture Notes

B. Sc. Mathematics (H)

I - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	1	2	2	**Ellipse-2**	**Dr. D. K. Yadav**	20

II - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	2	4	2	**Formation of Ordinary Differential Equations**	Dr. D. K. Yadav	22

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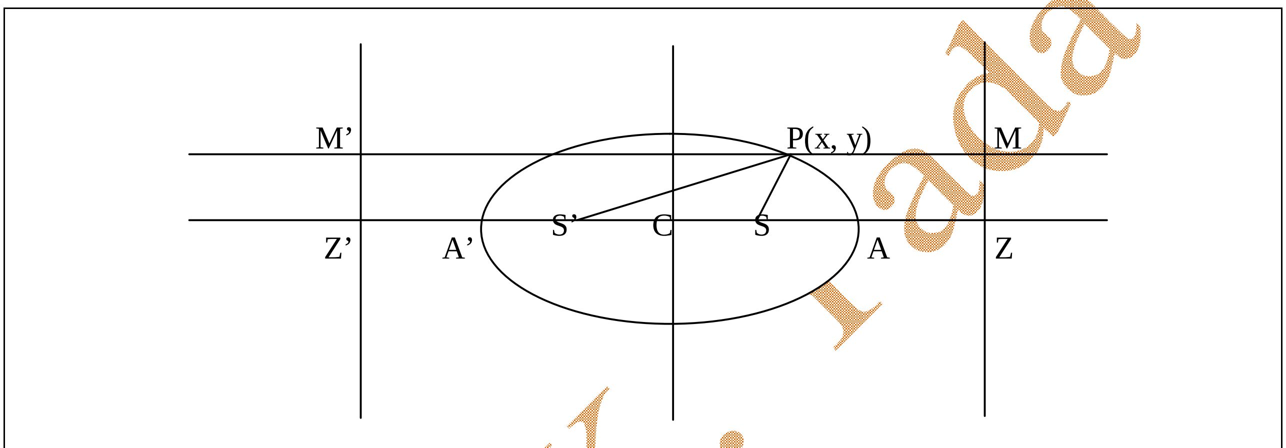
Part-1	**Part-2**
Paper-II	Paper-IV
B. Sc. Maths (H)	B. Sc. Maths (H)
Ellipse – 2	**Formation of Ordinary Differential Equations**

Ellipse – 2

Property: The sum of the focal distances of any point on an ellipse is constant and is equal to the length of the major axis of the ellipse.

Proof: Let P(x, y) is any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



From above figure we have

SP = e PM = e ZN (1)

S'P = e PM' = e Z'N (2)

By adding (1) and (2), we get

SP + SP' = e (CZ + CZ').

But CZ = a/e and CZ' = a/e. Therefore

SP + SP' = 2a = AA'

where AA' is the length of the major axis of the ellipse i.e., the sum of the focal distances of any point on an ellipse is constant and is equal to the length of the major axis of the ellipse.

Equation of Ellipse in other Forms:

(1) For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have

Centre = (0, 0)

Foci = (0, ae) and (0, -ae)

Equation of major axis is x = 0

Equation of minor axis is y = 0

Equation of directrices are y = a/e and y = -a/e

Latus Rectum = $2b^2/a$.

(2) For the ellipse $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$, we have

Centre = (α, β)

Foci = (ae + α, β) and $(-ae + \alpha, \beta)$

Equation of major axis is $y - \beta = 0$

Equation of minor axis is $x - \alpha = 0$

Equation of directrices are $x - \alpha$ = a/e and $x - \alpha$ = -a/e

Latus Rectum = $2b^2/a$.

Position of a point (h, k) with respect to Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

The point (h, k) is

outside the ellipse if $\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1 > 0$;

inside the ellipse if $\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1 < 0$;

on the ellipse if $\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1 = 0$.

Example: Find the equation of the ellipse whose one focus is at (2, 1) and the directrix is 2x – y + 3 = 0 and the eccentricity is $\frac{1}{\sqrt{2}}$.

Solution: Let us assume an arbitrary point (x, y) on the ellipse. Then by focus-directrix property

$$e = \frac{\text{distance of point from focus}}{\text{distance of point from directrix}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{(x-2)^2 + (y-1)^2}}{\frac{2x - y + 3}{\sqrt{(-2)^2 + (-1)^2}}}$$

which on simplification gives

$6x^2 + 4xy + 9y^2 - 52x - 14y + 41 = 0$

which is the required equation.

Formation of Ordinary Differential Equations

Formation of an Ordinary Differential Equation: Suppose we are given an equation containing n arbitrary constants. Then we differentiate it n times so as to get n additional equations containing the n arbitrary constants and derivatives. Now we eliminate n arbitrary constants from the above mentioned (n+1) equations and obtain an equation involving a derivative of nth order. Thus we form a differential equation of nth order.

Example: the differential equation for the family of curves $y=Ae^{2x}+Be^{-2x}$ is $y''=4y$.

Working Rule to Eliminate Arbitrary Constants:

Let us consider the linear equations

$$\begin{aligned} a_1x+b_1y+c_1&=0\\ a_2x+b_2y+c_2&=0\\ a_3x+b_3y+c_3&=0 \end{aligned}$$

Here we have to eliminate x and y. after eliminating these, we get an equation given by the determinant as

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

Example: Find the differential equation of the family of curves

$$y = Ae^{2x} + Be^{-2x}$$

for different values of A and B.

Solution: We have given that

$$y = Ae^{2x} + Be^{-2x} \qquad (1)$$

Differentiating it w.r.to x, we get

$$y' = 2Ae^{2x} - 2Be^{-2x} \qquad (2)$$

Again differentiating it w.r.to x we get

$$y'' = 4Ae^{2x} + 4Be^{-2x} = 4y \text{ [Using (1)]}$$

Thus the required differential equation is $y'' = 4y$.

Examples for Exercise

1. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$ for different values of A and B.

[Ans: $y'' = 4y$]

2. Find the differential equation of the family of curves $y = e^{x}(A\cos x + B\sin x)$ where A and B are arbitrary constants.

[Ans: $y'' - 2y' + 2y = 0$]

3. By eliminating the arbitrary constants a and b obtain the differential equation for which $xy = ae^{x} + be^{-x} + x^{2}$ is a solution.

[Ans: $xy'' + 2y' - xy + x^{2} - 2 = 0$]

4. Form the differential equation for y=Acos(nt)+Bsin(nt), a and b are parameters.

[Ans: $y'' + x^{2}y = 0$]