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Lecture Notes

B. Sc. Mathematics (H)

I - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	1	2	2	**Ellipse**	**Dr. D. K. Yadav**	19

II - Year

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	2	4	2	**Differential Equations**	Dr. D. K. Yadav	21

Dr. Dharmendra Kumar Yadav

Assistant Professor of Mathematics

M. L. S. College, Sarisab-Pahi

R. K. College, Madhubani

mdrdkyadav@gmail.com

+91-9891643856

Part-1	Part-2
Paper-II	Paper-IV
B. Sc. Maths (H)	B. Sc. Maths (H)
Ellipse	**Differential Equations**

Ellipse

An ellipse is a cone for which the eccentricity e is less than unity i.e. one. Therefore we can define ellipse as:

An ellipse is the locus of a point which moves in a plane so that the ratio of its distance from a fixed point to its distance from a fixed straight line in the same plane is a constant and is always less than unity (i.e. one).

Focus: The fixed point is called the focus of the ellipse.

Directrix: The fixed straight line is called the directrix of the ellipse.

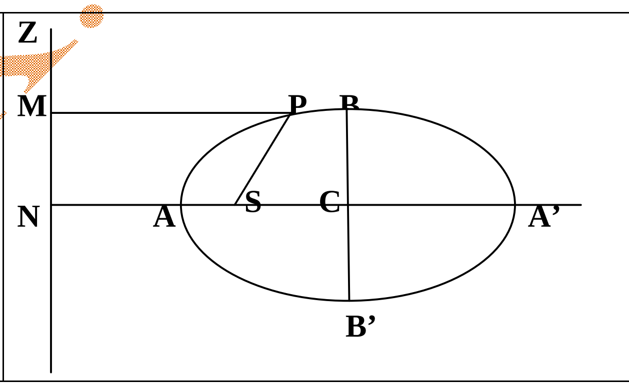
Eccentricity: The constant ratio (e) is called the eccentricity of the ellipse and for ellipse $e < 1$.

Shape of an Ellipse:

Let P be a point on an ellipse, the focus is the point S, and the directrix is ZN, such that PS < PM, where PM is perpendicular to ZN.

The point P must lie on the side of the directrix on which the focus S lies. Draw SN perpendicular to ZN.

Here $e = PS/PM$



Equation of an Ellipse:

Let (x, y) be any point the ellipse and e be the eccentricity. Let AA' be the 2a and BB' = 2b and C be the centre (origin) of AA' and BB' in the ellipse. Then the equation of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a, b and e are related by $b^2 = a^2 (1 - e^2)$.

For an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

we have the following formulae:

Centre: (0, 0)

Vertices: (a, 0) and (-a, 0)

Foci: S(ae, 0) and S'(-ae, 0) are the foci of the x-axis.

Axes: x = 0 and y = 0. AA' is major axis and BB' is minor axis.

Directrix: x + a/e = 0.

Ordinate, Double Ordinate, Latus Rectum:

From a point P on the ellipse, let PN be drawn which is perpendicular to the major axis AA' and produced to meet the curve again in P'. Then PN is called the ordinate and PNP' the double ordinate of the point P. A double ordinate through a focus is called a latus rectum and the ordinate through the focus a semi latus rectum.

$$\text{Latus Rectum} = \frac{2b^2}{a} = 2a(1 - e^2)$$

$$\text{Semi Latus Rectum} = \frac{b^2}{a} = a(1 - e^2)$$

Tracing the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

(i) The curve does not pass through the origin.

(ii) The curve meet the x-axis at (a, 0) and (-a, 0).

(iii) The curve meet the y-axis at (0, b) and (0, -b).

(iv) The curve is symmetric about both the axes.

(v) The curve don't lie either to the left of the line x = -a or to the right of the line x = a. The curve lies entirely between these two lines. Both the lines x = a and x = -a are tangents on the ellipse.

(vi) The curve don't extend either downwards of the line y = -b or upwards of the line y = b. The curve lies entirely between these two lines. Both the lines y = b and y = -b are tangents on the ellipse.

(vii) If x increases from 0 to a, y decreases from b to 0. Similarly if y increases from 0 to b, x decreases from a to 0.

These points are sufficient to draw an ellipse. The ellipse is a closed curve.

Example: Draw the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Differential Equations

Differential equation: An equation involving derivatives or differentials of one or more dependent variables with respect to one or more independent variables is called a differential equation.

Example:

$$dy=(x+\sin x)dx \qquad \ldots\ldots. (1)$$

$$\frac{d^4x}{dt^4}+\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^5=e^t \qquad \ldots\ldots (2)$$

Differential equations can be divided into two parts:

Ordinary Differential Equation: A differential equation involving derivatives with respect to a single independent variable is a called an ordinary differential equation.

Example:

$$y=\sqrt{x}\frac{dy}{dx}+\frac{k}{dy/dx} \qquad \ldots\ldots (3)$$

$$k\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} \qquad \ldots\ldots (4)$$

Partial Differential Equation: A differential equation involving partial derivatives with respect to more than one independent variable is called a partial differential equation.

Example:

$$\frac{\partial^2 v}{\partial t^2}=k\left(\frac{\partial^3 v}{\partial x^3}\right)^2 \qquad \ldots\ldots (5)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \qquad \ldots\ldots (6)$$

Order of a Differential Equation: The order of the highest order derivative involved in a differential equation is called the order of the differential equation.

Example: the equation (2) is of the fourth order, equations (1) and (3) are of first order, equations (4) and (6) are of the second order and equation (5) is of the third order.

Degree of a Differential Equation: The degree of a differential equation is the degree of the highest derivative which occurs in it, after the differential equation has been made free from radicals and fractions as far as the derivatives are concerned. This definition of degree does not require variables x, t, u etc. to be free from radicals and fractions.

Example: Differential equations (1), (2), and (6) are of first degree, equation (3) of second degree, equation (4) and (5) are of degree two.

Exercise

Find the order and degree of the following differential equations:

1. $(y'')^3 + y' = e^x$ [Ans: 2, 3]

2. $(y'')^2 + y''' = 4$ [Ans: 3, 1]

3. $y' + \frac{x}{y'} = K$ [Ans: 1, 2]

4. $y'' + yx = \sin y''$ [Ans: 2, undefined]

5. $y'' = (y')^{1/3}$ Ans: [2, 3]

6. $(2x+y)dx+(x-3y)dy=0$ [Ans: 1, 1]

Linear and Non-linear Differential Equations: A differential equation is called linear if (i) every dependent variable and every derivative involved occurs in the first degree only, and (ii) no product of dependent variables and/or derivatives occur. A differential equation which is not linear is called a non-linear differential equation.

Example: Equations (1) and (6) are linear and equations (2), (3), (4) and (5) are all non-linear.