Lecture Notes

B. Sc. Mathematics (H)

Sub	Year	Paper	Unit	Topic	Author	Lec. S. N.
Maths	1	1	2	Rings	**Dr. D. K. Yadav**	
Maths	2	3	2	Expansion of Functions	**Dr. D. K. Yadav**	

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Dr. Dharmendra Kumar Yadav

Assistant Professor of Mathematics

M. L. S. College, Sarisab-Pahi

R. K. College, Madhubani

Part-1

Paper-I

RINGS

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MACLAURIN'S SERIES EXPANSION

Part-1

Paper-I

B. Sc. Maths (H)

RINGS

Ring: Suppose R is a non-empty set equipped with two binary operations addition (+) and multiplication (.) i.e., for all a, b of R we have (a + b) and (a . b) belong to R. Then the algebraic structure (R, +, .) is called a ring, if the following postulates are satisfied:

(R, +) is an Abelian Group:

R-1: Addition is Closure: $a + b \in R$ for all a, b $\in$ R.

R-2: Addition is Associative: (a + b) + c = a + (b + c) for all a, b, c $\in$ R.

R-3: Existence of Additive Identity: There exists an additive identity 0 $\in$ R such that

a + 0 = a = 0 + a, for all a $\in$ R.

R-4: Existence of Additive Inverse: There exists an additive inverse -a $\in$ R such that

a + (-a) = 0 = (-a) + a, for all a $\in$ R.

R-5: Addition is Commutative: a + b = b + a, for all a, b $\in$ R.

(R, .) is a Semi Group:

R-6: Multiplication is Closure: $a.b \in R$ for all a, b $\in$ R.

R-7: Multiplication is Associative: (a . b) . c = a . (b . c) for all a, b, c $\in$ R.

Multiplication is Distributive over Addition:

R-8: a . (b + c) = a . b + a . c, for a, b, c $\in$ R. [Left Distributive law]

R-9: (b + c) . a = b . a + c . a, for a, b, c $\in$ R. [Right Distributive law]

Zero Element: The element 0 the identity element for addition operation in R, is called the zero element of the ring R.

Every ring possesses a unique zero element and it will be the identity element for addition operation. We shall always denote the zero element by the symbol 0.

Note:

1. The equation a + x = b has a unique solution in R and is given by x = b – a. Similarly the equation y + a = b has a unique solution y = b – a in R.

2. If in a ring R we have a + b = 0, then a = - b and b = - a.

Ring with Unity: If in a ring R there exists an element denoted by 1 such that 1.a = a = a.1 for all a of R, then R is called a ring with unit element.

The element 1$\in$R is called the unit element of the ring R. obviously 1 is the multiplicative identity of R.

Thus if a ring possesses multiplicative identity, then it is a ring with unity.

Commutative Ring: If in a ring R, the multiplication operation is also commutative i.e., if we have a . b = b . a for all a, b $\in$R, then R is called a commutative ring.

Elementary Properties of a Ring:

If R is a ring, then for all a, b, c $\in$ R, we have

(i) $a \cdot 0 = 0 \cdot a = 0$

(ii) $a \cdot (-b) = -(a \cdot b) = (-a) \cdot b$

(iii) $(-a) \cdot (-b) = a \cdot b$

(iv) $a(b - c) = a \cdot b - a \cdot c$

(v) $(b - c)a = b \cdot a - c \cdot a$

Examples:

1. The set I of all integers is a ring with respect to addition and multiplication of integers as the two ring operations. This ring is called the **ring of integers**. This is a commutative ring with unity.

2. The set Q of all rational numbers is a commutative ring with unity.

3. The set R of all real numbers is a commutative ring with unity.

4. The set C of all complex numbers is a commutative ring with unity.

Zero Divisor:

A non-zero element 'a' of a ring R is called a zero divisor or a divisor of zero if there exists an element $b \neq 0$ in R such that either a.b = 0 or b.a = 0.

Ring without Zero Divisors:

A ring R is called a ring without zero divisors if the product of no two non-zero elements of R is zero i.e., a.b = 0 implies that either a = 0 or b = 0.

Ring with Zero Divisors:

A ring R is called a ring with zero divisors if there exists two non-zero elements a, b in R such that their product is zero i.e., a.b = 0.

Example: The ring of integers is a ring without zero divisors.

Cancellation Laws in a Ring:

If a, b, c are elements of a ring R and

1. If $a \neq 0$, then a.b = a.c implies b = c.

2. If $a \neq 0$, then b.a = c.a implies b = c.

Theorem: A ring R is without zero divisors if and only if the cancellation laws hold in R.

Inversible Elements in a Ring with Unity:

In a ring every element possesses additive inverse. Therefore the question of an element being inversible or not arises only with respect to multiplication.

If R is a ring with unity, then an element a of R is called inversible, if there exists b in R such that a.b = 1 = b.a. Also then we write $b = a^{-1}$.

Examples:

(1) 1 and -1 are the only two inversible elements of the ring of all integers.

(2) n x n non-singular matrices with real numbers as elements are the only inversible elements of the ring of all n x n matrices with elements as real numbers.

Boolean Ring:

A ring R is called a Boolean Ring if all its elements are idempotent i.e., $a^2 = a$ for all a of R.

Part-2

Paper-III

B. Sc. Maths (H)

MACLAURIN'S SERIES EXPANSION

Maclaurin's Theorem:

If a function f(x) is differentiable any number of times and can be expanded as an infinite convergent series of terms of positive integral powers of x, then

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + ... + \frac{x^n}{n!} f^n(0) + ... = \sum_{n=0}^{\infty} \frac{x^n}{n!} f^n(0)$$

where $f^n(0)$ stands for nth derivative of f(x) at x=0. The series on the R.H.S. is known as Maclaurin's series or Maclaurin's expansion of f(x).

Conditions Under Which the Maclaurin's Expansion is Valid:

(1) f(x) and its successive derivatives must be finite and continuous in the range of x in which f(x) is defined.

(2) The series on the right hand side must be convergent.

For the condition of convergence of the power series on R.H.S., if we write

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + ... + \frac{x^n}{n!} f^n(0) + R_n$$

Then, the remainder $R_n \to 0$ as $n \to \infty$.

Failure of Maclaurin's Theorem:

Every function cannot be expanded by Maclaurin's theorem. This theorem is not applicable in the following cases:

(1) The function f(x) or any of its successive derivatives do not exist finitely at x=0.

(2) The infinite series obtained by expansion does not converge.

Example: Maclaurin's theorem cannot be applied to obtain the expansion of functions like $\cot x$, $\log x$, etc

Note: The Lagrangian form of the remainder R_n is $R_n = \frac{x^n}{n!} f^n(\theta x)$; $0 < \theta < 1$.

Example: Expand the following functions in ascending power of x:

(1) Sinx

(2) Cosx

(3) log(1+x)

(4) e^x