Lecture Notes

B. Sc. Maths (H)

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Part-1

Paper-I

B. Sc. Maths (H)

CENTRE OF A CONIC SECTION

Part-2

Paper-III

B. Sc. Maths (H)

CONVERGENCY AND DIVERGENCY OF INFINITE SERIES

Part-1

Paper-I

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CENTRE OF A CONIC SECTION

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Centre of a Conic Section

The centre of a conic section is that point at which all chords of the conic drawn through it are bisected.

Let (x_1, y_1) be the centre on the conic and the general equation of the conic is given by

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Then the centre is given by

$$(x_1, y_1) = \left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right)$$

Equation of Conic Section Referred to the Centre of the Conic as Origin

Let the equation of the conic be

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Let (α, β) be the centre. Then the equation of the conic referred to the centre as origin is

$$ax^2 + 2hxy + by^2 + \frac{abc + 2fgh - af^2 - bg^2 - ch^2}{ab - h^2} = 0$$

Example:

1. Find the co-ordinates of the centre of the conic

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

and its equation referred to the centre as origin.

2. What conic section is represented by

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \ ?$$

Find the centre, axes and eccentricity. [LNMU 2019 (H)]

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Part-2

Paper-III

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CONVERGENCY AND DIVERGENCY OF INFINITE SERIES

Logarithmic Test: If for the positive term series $\sum U_n$,

$$\lim_{n \to \infty}\left(n \log \frac{U_n}{U_{n+1}}\right) = \lambda .$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda > 1$,

(2) Divergent if $\lambda < 1$, and

(3) Test fails if $\lambda = 1$.

Note: When $\frac{U_n}{U_{n+1}}$ involves e, we apply logarithmic test after the Ratio test.

Higher Logarithmic Test: If for the positive term series $\sum U_n$,

$$\lim_{n \to \infty}\left[\left\{\left(n \log \frac{U_n}{U_{n+1}}\right) - 1\right\}\log n\right] = \lambda .$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda > 1$,

(2) Divergent if $\lambda < 1$, and

(3) Test fails if $\lambda = 1$.

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Note: Generally this test is applied when Logarithmic test fails.

Example: Find whether the series

$$x + x^{1+\frac{1}{2}} + x^{1+\frac{1}{2}+\frac{1}{3}} + x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} + \cdots$$

is convergent or divergent.

[Hint: Convergent, when x < (1/e) and divergent, if x ≥ (1/e).]

Cauchy's Root Test: If for the positive term series $\sum U_n$,

$$\lim_{n \to \infty} (U_n)^{1/n} = \lambda .$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda < 1$,

(2) Divergent if $\lambda > 1$, and

(3) Test fails if $\lambda = 1$.

Example: Test the convergence of the series whose general terms is given by

$$\frac{n^{n^2}}{(1+n)^{n^2}}$$

[Hint: Convergent]

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Gauss Test: If for the series $\sum U_n$ of positive terms, $\frac{U_n}{U_{n+1}}$ can be expanded in the form

$$\frac{U_n}{U_{n+1}} = 1 + \frac{\lambda}{n} + O\left(\frac{1}{n^2}\right)$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda > 1$, and

(2) Divergent if $\lambda \leq 1$.

Note: This test never fails as we know that the series diverges for $\lambda = 1$. Moreover the test is applied after the failure of Ratio test and when it is possible to expand $\frac{U_n}{U_{n+1}}$ in powers of $\frac{1}{n}$ by Binomial theorem or by other method.

Example: Prove by applying Gauss's test that the series

$$\frac{1^2}{2^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \cdot \frac{5^2}{6^2} + \cdots$$

divergent.

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