Lecture Notes

B. Sc. Maths (H)

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Part-1

Paper-I

B. Sc. Maths (H)

EQUATION OF NORMAL ON CONIC SECTION

Part-2

Paper-III

B. Sc. Maths (H)

CONVERGENCY AND DIVERGENCY OF INFINITE SERIES

Part-1

Paper-I

B. Sc. Maths (H)

EQUATION OF NORMAL ON CONIC SECTION

Normal at a Point of a Conic Section

The normal to a conic at a point of it is the line perpendicular to the tangent to the conic at that point.

Let the equation of the conic is

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

We know that the 'm' of the tangent is given by

$$= \frac{dy}{dx} = -\frac{\frac{\partial S}{\partial x}}{\frac{\partial S}{\partial y}}$$

Therefore the 'm' of the normal is

$$= -\frac{1}{\frac{dy}{dx}} = \frac{\frac{\partial S}{\partial y}}{\frac{\partial S}{\partial x}}$$

The equation of the normal at the point (x_1, y_1) is

$$y - y_1 = \frac{\left(\frac{\partial S}{\partial y}\right)_{(x_1,y_1)}}{\left(\frac{\partial S}{\partial x}\right)_{(x_1,y_1)}}(x - x_1)$$

Putting the values of partial derivatives in it, we get the equation of the normal as

$$\frac{(x - x_1)}{ax_1 + hy_1 + g} = \frac{(y - y_1)}{hx_1 + by_1 + f}$$

Particular Cases of Equation of Normal for Conic Sections:

(a) The equation of the normal to the parabola $y^2 = 4ax$ at (x_1, y_1) is

$$\frac{(x - x_1)}{-2a} = \frac{(y - y_1)}{y_1}$$

(b) The equation of normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is

$$\frac{(x - x_1)}{\frac{x_1}{a^2}} = \frac{(y - y_1)}{\frac{y_1}{b^2}}$$

(c) The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is

$$\frac{(x - x_1)}{\frac{x_1}{a^2}} = \frac{(y - y_1)}{-\frac{y_1}{b^2}}$$

Theorem on Three Normals:

Prove that from a point (x1, y1) there are three normals to a parabola $y^2 = 4ax$.

[Do Yourself from Book]

Example: Find the equation of the normal to the parabola $y^2 = 8x$ at the point (2, 4) and prove that the normal at the point (2, 4) meets the parabola again at the point (18, -12).

Part-2

Paper-III

B. Sc. Maths (H)

CONVERGENCY AND DIVERGENCY OF INFINITE SERIES

D' Alembert's Ratio Test (or Ratio Test): If for the positive term series $\sum U_n$,

$$\lim_{n \to \infty} \frac{U_n}{U_{n+1}} = \lambda .$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda > 1$,

(2) Divergent if $\lambda < 1$, and

(3) Test fails if $\lambda = 1$.

Note: When this test fails, apply Comparison test or the test for divergence that $\lim_{n \to \infty} U_n \neq 0$

Example: Test the convergency of the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \ldots..$$

[Hint: Convergent for x ≤ 1 and divergent for x > 1]

Higher Ratio Test (Raabe's Test): If $\sum U_n$ be the given positive term series such that

$$\lim_{n \to \infty} \left\{ n\left(\frac{U_n}{U_{n+1}} - 1 \right) \right\} = \lambda .$$

Then, the given series $\sum U_n$ is

(1) Convergent if $\lambda > 1$,

(2) Divergent if $\lambda < 1$, and

(3) Test fails if $\lambda = 1$.

Note: Generally this test is applied when D'Alembert's ratio test fails. This method is directly applied if $\frac{U_n}{U_{n+1}}$ is a fraction of two polynomials, such that the two polynomials have the same degree and the coefficients of the highest powers of n in both numerator and denominator are equal. This test is used when $\frac{U_n}{U_{n+1}}$ does not involve the number e. When $\frac{U_n}{U_{n+1}}$ involves e, we apply logarithmic test after the Ratio test and not Raabe's test.

Example: Test the convergence of the series

$$\frac{2}{3.4} + \frac{2.4}{3.5.6} + \frac{2.4.6}{3.5.7.8} + \cdots + \frac{2.4.6 \ldots 2n}{3.5.7 \ldots (2n+1).(2n+2)} + \cdots$$

[Hint: Convergent]

De Morgan and Bertrand's Test: If $\sum U_n$ be the given positive term series such that

$$\lim_{n \to \infty} \left[\left\{ n\left(\frac{U_n}{U_{n+1}} - 1 \right) - 1 \right\} \log n \right] = \lambda .$$

Then, the series $\sum U_n$ is

(1) Convergent if $\lambda > 1$,

(2) Divergent if $\lambda < 1$, and

(3) Test fails if $\lambda = 1$.

Note: Generally this test is applied when Raabe's test fails. This method is directly applied if $n\left(\frac{U_n}{U_{n+1}} - 1\right)$ is a fraction of two polynomials, such that the two polynomials have the same degree and the coefficients of the highest powers of 'n' in both numerator and denominator are equal.

Example: Test the convergence of the series

$$\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \cdots$$

[Hint: Divergent]